MATH 2230 A Complex Variables with Application

Suggested Solution for HW5

1. Proof: When the branch $\log z = \ln r + i\theta$ $(r>0, \frac{3}{4}\pi < \theta < \frac{11}{4}\pi)$ is used,

$\log(i^2) = \log(-1) = \ln 1 + i\pi = \pi i$

$2\log i = 2(\ln 1 + i\frac{5}{2}\pi) = 5\pi i$

Thus, $\log(i^2) \neq 2\log i$

2. Proof: (a) $i = e^{i(\frac{\pi}{2}+2k\pi)}$ $\quad k = 0, \pm 1, \pm 2, \cdots$

$C_k = e^{i(\frac{\pi}{4}+k\pi)}$ $\quad k = 0, 1$

Thus two roots of i are $C_0 = e^{\frac{i\pi}{4}}$ and $C_1 = e^{i\frac{5}{4}\pi}$

$\log(e^{i\frac{\pi}{4}}) = \ln 1 + i(\frac{\pi}{4} + 2n\pi)$ $\quad (n = 0, \pm 1, \pm 2, \cdots)$

$= (2n + \frac{1}{4})\pi$ $\quad (n = 0, \pm 1, \pm 2, \cdots)$

$\log(e^{i\frac{5}{4}\pi}) = \ln 1 + i(\frac{5}{4}\pi + 2n\pi)$ $\quad (n = 0, \pm 1, \pm 2, \cdots)$

$= [(2n+1) + \frac{1}{4}]\pi i$ $\quad (n = 0, \pm 1, \pm 2, \cdots)$

Thus, $\log(i^{\frac{1}{2}}) = (n + \frac{1}{4})\pi i$ $\quad (n = 0, \pm 1, \pm 2, \cdots)$

(b) $\frac{1}{2}\log i = \frac{1}{2}(\ln 1 + i(\frac{\pi}{2} + 2n\pi))$

$= (\frac{1}{4} + n)\pi i$ $\quad (n = 0, \pm 1, \pm 2, \cdots)$

$= \log(i^{\frac{1}{2}})$

3. Proof: $\log(e^z) = \ln|e^z| + i(y + 2n\pi)$

$= \ln e^x + i(y + 2n\pi)$

$= x + i(y + 2n\pi)$ $\quad (n = 0, \pm 1, \pm 2, \cdots)$

When the branch $\log z = \ln r + i\theta$ $(r > 0, \alpha < \theta < \alpha + 2\pi)$ is used,

$\alpha < y + 2n\pi < \alpha + 2\pi$

Noted $\alpha < y < \alpha + 2\pi$, we have $n = 0$

$\log(e^z) = x + iy = z$